Applied Quantum Field Theory

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2 Participants

2.1 ICRANet participants

- Hagen Kleinert
- Remo Ruffini
- She-Sheng Xue

2.2 Ongoing collaborations

- Alexander Chervyakov (FU-Berlin, Germany)
- Patrick Haener (Nomura Bank, London, Great Britain)
- Petr Jizba (FU-Berlin, Germany)
- Flavio Nogueira (FU-Berlin, Germany)
- Axel Pelster (Universität Essen-Duisburg, Germany)

2.3 Students

- Tim X.J. Chen (FU-Berlin, Germany)
- Konstantin Glaum (FU-Berlin, Germany)
- Sonja Overesch (FU-Berlin, Germany)
- Walja Korolevski (FU-Berlin, Germany)
- Mathias Ohlinger (FU-Berlin, Germany)
- Moritz Schütte (FU-Berlin, Germany)
- Steffen Röthel (FU-Berlin, Germany)
- Matthias Ohliger (FU-Berlin, Germany)
- Pascal Mattern (FU-Berlin, Germany)

- Barry Bradlyn (FU-Berlin, Germany)
- Kiel Howe (FU-Berlin, Germany)
- Ednilson Santos (FU-Berlin, Germany)
- Alexander Hoffmann (FU-Berlin, Germany)

3 Brief Description

This subgroup of ICRANet is occupied with various physical fields. Most closely related to Astrophysics is the collaboration with Ruffini and Xue (1), where we investigate the different rates of pair production in space-dependent electric fields. In a subsequent work with Alexander Chervyakov (2)

The work on phase transition (3; 4) is linked to astrophysics by the possible phase transitions in neutron stars. In this context the question whether there exists color superconductivity in nuclear matter at high pressure will ultimately have to be answered.

The work on Bose-Einstein condensation (5; 6) is preparatory to ultimately understand the problem of stability of Bose stars, tackled by Ruffini and Bonozzola many years ago (7).

The work on the relation between geometry and defects is of interest on fundamental and pedagogical grounds. On the one hand, we have developed the most pedagogical method of teaching the properties of spacetimes with curvature and torsion (Einstein-Cartan spacetimes). On the other hand we have discovered a new principle according to which the physics in spaces with curvature and torsion can be predicted from the well-known physical laws in flat spacetime by performing simply a multivalued coordinate transformation. This technique is successful not only in the realm of gravity (8), but also in the theory of vortex lines in superfluids and the theory of plastic deformation of materials, as explained in the textbook (9). From the work done in this project, a new textbook (4) has emerged.

The work on econophysics serves to understand fluctuating systems by methods developed in quantum physics. Here we continue to deduce practical consequences of the methods developed in the textbook (6) for financial markets. In this context it was essential to derive the analog of It's formula for non-Gaussian fluctuations and to find from it techniques how to stabilize fluctuating portfolios of investment (hedging) (10).

4 Publications (2005 - 2008)

1. H. Kleinert
"*Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets*" World Scientific, Singapore 2006, pp. 1-1547

2. B. Hamprecht and H. Kleinert "*End-To-End Distribution Function Function of Stiff Polymers for all Persistence Lengths* " Phys. Rev. E **71**, 031803 (2005) (cond-mat/0305226)

   We set up recursion relations for calculating all even moments of the end-to-end distance of a Porod-Kratky wormlike chains in D dimensions. From these moments we derive a simple analytic expression for the end-to-end distribution in three dimensions valid for all peristence lengths. It is in excellent agreement with Monte Carlo data for stiff chains and goes properly over into the Gaussian random-walk distributions for low stiffness.

3. H. Kleinert and V.I. Yukalov "*Highly Accurate Critical Exponents from Self-Similar Variational Perturbation Theory* " Phys. Rev. E **71**, 026131 (2005) (cond-mat/0402163)

   We extend field theoretic variational perturbation theory by self-similar approximation theory, which greatly accelerates convergence. This is illustrated by re-calculating the critical exponents of O(N)-symmetric ϕ^4 theory. From only three-loop perturbation expansions in $4-\epsilon$ dimensions we obtain *analytic results for the exponents, with practically the same accuracy as those derived recently from ordinary field-theoretic variational perturbational theory to seventh order. In particular, the theory explains the best-measured exponent* $\alpha \approx -0.0127$ *of the specific heat peak in superfluid helium, found in a satellite experiment with a temperature resolution of nanoKelvin. In addition, our analytic expressions reproduce also the exactly known large-N behaviour of the exponents* ν *and* $\gamma = \nu(2-\eta)$ *with high precision.*

4. S.F. Brandt, H. Kleinert, A. Pelster "*Recursive Calculation of Effective Potential and Variational Resummation* " J. Math. Phys. **46**, 032101 (2005) (quant-ph/0406206)

   We set up a method for a recursive calculation of the effective potential which is applied to a cubic potential with imaginary coupling. The result is resummed using variational perturbation theory (VPT), yielding an exponentially fast convergence.

5. O. Zobay, G. Metikas, H. Kleinert "*Nonperturbative Effects on T_c of Interacting Bose Gases in Power Traps* " Phys. Rev. A **71**, 043614 (2005) (cond-mat/0411133)

   The critical temperature T_c of an interacting Bose gas trapped in a general power-law potential $V(x) = \sum_i U_i |x_i|^{p_i}$ is calculated with the help of variational perturbation theory. It is shown that the interaction-induced shift in T_c fulfills the relation $(T_c - T_c^0)/T_c^0 = D_1(eta)a + D'(eta)a^{2eta} + O(a^2)$ with T_c^0 the critical temperature of the trapped ideal gas, a the s-wave scattering length divided by the thermal wavelength at T_c, and $\eta = 1/2 + \sum_i 1/p_i$ the potential-shape parameter. The terms $D_1(\eta)a$ and $D'(\eta)a^{2\eta}$ describe the leading-order perturbative and nonperturbative contributions to the critical temperature, respectively. This result quantitatively shows how an increasingly inhomogeneous potential suppresses the influence of critical fluctuations. The appearance of the $a^{2\eta}$ contribution is qualitatively explained in terms of the Ginzburg criterion.

6. H. Kleinert "*Order of Superconductive Phase Transition* " Berlin Preprint 2005 publ. in Festschrift in honor of R. Folk's 60th birthday (2004).

   On the occasion of Reinhard Folk's 60th birthday, I give a brief review of the theoretical progress in understanding the critical properties of superconductors. I point out the theoretical difficulties in finding a second-order transition in the Ginzburg-Landau Model with O(N)-symmetry in 4-e Dimensions, and the success in predicting the existence and location of a tricritical point with the help of a dual disorder theory.

7. M. Blasone, P. Jizba, and H. Kleinert "*Quantum Behavior of Deterministic Systems with Information Loss. Path Integral Approach* " Phys. Rev. A **71** 052507 (2005) (quant-ph/0409021)

   We present a path-integral formulation of 't Hooft's derivation of quantum from classical physics. The crucial ingredient of this formulation is Gozzi et al.'s supersymmetric path integral of classical mechanics. We quantize explicitly two simple classical systems: the planar mathematical pendulum and the Roessler dynamical system.

8. H. Kleinert and A.J.M. Schakel "*Anomalous Dimension of Dirac's Gauge-Invariant Nonlocal Order Parameter in Ginzburg-Landau Field Theory* " Phys. Lett. B **611**, 182 (2005)

   In a Ginzburg-Landau theory with n fields, the anomalous dimension of the gauge-invariant nonlocal order parameter defined by the long-distance limit of Dirac's gauge-invariant two-point function is calculated. The result is exact for all n to first order in $\epsilon \equiv 4 - d$, and for all $d \in (2, 4)$ to first order in $1/n$, and coincides with the previously calculated gauge-dependent exponent in the Landau gauge.

9. H. Kleinert, S. Schmidt, and A. Pelster "*Quantum Phase For Homogeneous Bose-Einstein Condensate* " Ann. Phys. **14**, 214 (2005) (cond-mat/0308561)

   We calculate the quantum phase transition for a homogeneous Bose gas in the plane of s-wave scattering length a_s and temperature T. This is done by improving a one-loop result near the interaction-free Bose-Einstein critical temperature $T_c^{(0)}$ with the help of recent high-loop results on the shift of the critical temperature due to a weak atomic repulsion using variational perturbation theory. The quantum phase diagram shows a nose above $T_c^{(0)}$, so that we predict the existence of a reentrant transition above $T_c^{(0)}$, where an increasing repulsion leads to the formation of a condensate.

10. H. Kleinert and A. Chervyakov "*Perturbation Theory for Path Integrals of Stiff Polymers* " J. Phys. A: Math. Gen. 39 8231 (2006) (cond-mat/0503199)

    The wormlike chain model of stiff polymers is a nonlinear σ-model in one spacetime dimension in which the ends are fluctuating freely. This causes important differences with respect to the presently available theory which exists only for periodic and Dirichlet boundary conditions. We modify this theory appropriately and show how to perform a systematic large-stiffness expansions for all physically interesting quantities in powers of L/ξ, where L is the length and ξ the persistence length of the polymer. This requires special procedures for regularizing highly divergent Feynman integrals which we have developed in previous work. We show that by adding to the unperturbed action a correction term $\mathcal{A}^{\rm corr}$, we can calculate all Feynman diagrams with Green functions satisfying Neumann boundary conditions. Our expansions yield, order by order, properly normalized end-to-end distribution function in arbitrary dimensions d, its even and odd moments, and the two-point correlation function.

11. H. Kleinert "*Emerging Gravity from Defects in World Crystal*
    Lecture Presented at the 2004 Conference on Emerging Gravity in Piombino Braz. J. Phys. **35**, 359 (2005)

    I show that Einstein Gravity can be thought of as arising from the defects in a world crystal whose lattice spacing is of the order of the Planck length $l_P \sim 10^{-33}$ cm, and whose elastic energy is of the second-gradient type (floppy crystal). No physical experiment so far would be able to detect the lattice structure.

12. H. Kleinert "*Vortex Origin of Tricritical Point in Ginzburg-Landau Theory* "Europhys. Letters **74**, 889 (2006) (cond-mat/0509430) Motivated by recent experimental progress in the critical regime of high-T_c superconductors we show how the tricritical point in a superconductor can be derived from the Ginzburg-Landau theory as a consequence of vortex fluctuations. Our derivation explains why usual renormalization group arguments always produce a

first-order transition, in contrast to experimental evidence and Monte Carlo simulations.

13. F.S. Nogueira and H. Kleinert "*Quantum Electrodynamics in* $2+1$ *Dimensions, Confinement, and the Stability of* $U(1)$ *Spin Liquids* " Phys. Rev. Lett. **95**, 176406 (2005) (cond-mat/0501022)

    Compact quantum electrodynamics in 2+1 dimensions often arises as an effective theory for a Mott insulator, with the Dirac fermions representing the low-energy spinons. An important and controversial issue in this context is whether a deconfinement transition takes place. We perform a renormalization group analysis to show that deconfinement occurs when $N > N_c = 36/\pi^3 \approx 1.161$, where N is the number of fermion replica. For $N < N_c$, however, there are two stable fixed points separated by a line containing a unstable non-trivial fixed point: a fixed point corresponding to the scaling limit of the non-compact theory, and another one governing the scaling behavior of the compact theory. The string tension associated to the confining interspinon potential is shown to exhibit a universal jump as $N \to N_c^-$. Our results imply the stability of a spin liquid at the physical value N=2 for Mott insulators.

14. M. Blasone, P. Jizba, and H. Kleinert "*Quantum Behavior of Deterministic Systems with Information Loss. Path Integral Approach* "Annals Phys. **320**, 468 (2005) (quant-ph/0504200)

    't Hooft's derivation of quantum from classical physics is analyzed by means of the classical path integral of Gozzi et al.. It is shown how the key element of this procedure - the loss of information constraint - can be implemented by means of Faddeev-Jackiw's treatment of constrained systems. It is argued that the emergent quantum systems are identical with systems obtained in [Phys.Rev. A 71 (2005) 052507] through Dirac-Bergmann's analysis. We illustrate our approach with two simple examples - free particle and linear harmonic oscillator. Potential Liouville anomalies are shown to be absent.

15. V.I. Yukalov and H. Kleinert "*Gapless Hartree-Fock-Bogoliubov Approximation for Bose Gas* " Phys. Rev. A **73**, 063612 (2006) (cond-mat/0606484)

    A dilute Bose system with Bose-Einstein condensate is considered. It is shown that the Hartree-Fock-Bogolubov approximation can be made both conserving as well as gapless. This is achieved by taking into account all physical normalization conditions, that is, the normalization condition for the condensed particles and that for the total number of particles. Two Lagrange multipliers, introduced for preserving these normalization conditions, make the consideration completely self-consistent.

16. F.S. Nogueira and H. Kleinert "*Thermally Induced Rotons in Two-Dimensional Dilute Bose Gases* "Phys. Rev. **73**, 104515 (2006) (cond-mat/0503523)

We show that roton-like excitations are thermally induced in a two-dimensional dilute Bose gas as a consequence of the strong phase fluctuations in two dimensions. At low momentum, the roton-like excitations lead for small enough temperatures to an anomalous phonon spectrum with a temperature dependent exponent reminiscent of the Kosterlitz-Thouless transition. Despite the anomalous form of the energy spectrum, it is shown that the corresponding effective theory of vortices describes the usual Kosterlitz-Thouless transition. The possible existence of an anomalous normal state in a small temperature interval is also discussed.

17. H. Kleinert and X.J. Chen "*Boltzmann Distribution and Market Temperature* " Berlin preprint 2006 (physics/0609209)

    The minute fluctuations of of S&P 500 and NASDAQ 100 indices display Boltzmann statistics over a wide range of positive as well as negative returns, thus allowing us to define a *market temperature* for either sign. With increasing time the sharp Boltzmann peak broadens into a Gaussian whose volatility σ measured in $1/\sqrt{\text{min}}$ is related to the temperature T by $T = \sigma/\sqrt{2}$. Plots over the years 1990–2006 show that the arrival of the 2000 crash was preceded by an increase in market temperature, suggesting that this increase can be used as a warning signal for crashes. A plot of the Dow Jones temperature over 78 years reveals a remarkable stability through many historical turmoils, interrupted only by short heat bursts near the crashes.

18. H. Kleinert "*Stiff Quantum Polymers* "Phys. Rev. B 76, 052202 (2007), Berlin preprint 2006

    At ultralow temperatures, polymers exhibit quantum behavior, which is calculated here for the second and fourth moments of the end-to-end distribution in the large-stiffness regime. The result should be measurable for polymers in wide optical traps.

19. H. Kleinert "*Field Transformations and Multivalued Fields* " 2007 J. Phys.: Conf. Ser. 67 012007 , Berlin preprint 2006

    Changes of field variables may lead to multivalued fields which do not satisfy the Schwarz integrability conditions. Their quantum field theory needs special care as is shown in an application to the superfluid and superconducting phase transitions.

20. F. Nogueira and H. Kleinert "*Compact quantum electrodynamics in* $2+1$ *dimensions and spinon deconfinement: a renormalization group analysis* " Berlin preprint 2007 (arXiv:0705.3541)

    We discuss compact (2+1)-dimensional Maxwell electrodynamics coupled to fermionic matter with N replica. For large enough N, the latter corresponds to an effective theory for the nearest neighbor SU(N) Heisenberg antiferromagnet, in which the fermions represent solitonic excitations known as spinons.

Here we show that the spinons are deconfined for $N > N_c = 36$, thus leading to an insulating state known as spin liquid. A previous analysis considerably underestimated the value of N_c. We show further that for $20 < N \leq 36$ there can be either a confined or a deconfined phase, depending on the instanton density. For $N \leq 20$ only the confined phase exist. For the physically relevant value N=2 we argue that no paramagnetic phase can emerge, since chiral symmetry breaking would disrupt it. In such a case a spin liquid or any other nontrivial paramagnetic state (for instance, a valence-bond solid) is only possible if doping or frustrating interactions are included.

21. J.W. Zhang, Y. Zhang, and H. Kleinert "*Power tails of Index Distributions in Chinese Stock Market* " Berlin preprint 2007

    The power a of the Lvy tails of stock market fluctuations discovered in recent years are generally believed to be universal. We show that for the Chinese stock market this is not true, the powers depending strongly on anomalous daily index changes short before market closure, and weakly on the opening data.

22. K. Glaum, A. Pelster, H. Kleinert, and T. Pfau "*Critical Temperature of Weakly Interacting Dipolar Condensates* " Phys. Rev. Lett. **98**, 080407/1-4 (2007) (cond-mat/0606569)

    We calculate perturbatively the effect of a dipolar interaction upon the Bose-Einstein condensation temperature. This dipolar shift depends on the angle between the symmetry axes of the trap and the aligned atomic dipole moments, and is extremal for parallel or orthogonal orientations, respectively. The difference of both critical temperatures exhibits most clearly the dipole-dipole interaction and can be enhanced by increasing both the number of atoms and the anisotropy of the trap. Applying our results to chromium atoms, which have a large magnetic dipole moment, shows that this dipolar shift of the critical temperature could be measured in the ongoing Stuttgart experiment.

23. H. Kleinert and S.-S. Xue Photoproduction in *Semiconductors by Onset of Magnetic Field Eur. Phys. Letters* 81, 57001 (2008).

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