Cosmology and Large Scale Structures

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3 Brief description

3.1 Numerical method and simulation of cosmological study

3.1.1 WENO scheme of numerical cosmological hydrodynamics

Introduction of WENO scheme

WENO schemes are a class of high order numerical methods for solving partial differential equations whose solutions may contain discontinuities, sharp gradient regions, and other complex solution structures. Such partial differential equations include the Euler equations or high Reynolds number Navier-Stokes equations which appear often in astronomical applications. WENO schemes are designed based on the successful essentially non-oscillatory (ENO) schemes in Harten, Engquist, Osher and Chakravarthy (46) and Shu and Osher (108; 109). The first WENO scheme was constructed in Liu, Osher and Chan (69) for a third order finite volume version in one space dimension. In Jiang and Shu (54), third and fifth order finite difference WENO schemes in multi space dimensions are constructed, with a general framework for the design of the smoothness indicators and nonlinear weights. Later developments and applications of WENO schemes mostly follow the approach in (54). Very high order finite difference WENO schemes (for orders between 7 and 11) have been developed in Balsara and Shu (3). Finite volume WENO schemes for 2D general triangulation have been developed in Friedrichs (34) and in Hu and Shu (50).

Both ENO and WENO schemes use the idea of adaptive stencils in the reconstruction procedure based on the local smoothness of the numerical solution to automatically achieve high order accuracy and non-oscillatory property near discontinuities. ENO uses just one (optimal in some sense) out of many candidate stencils when doing the reconstruction; while WENO uses a convex combination of all the candidate stencils, each being assigned a nonlinear weight which depends on the local smoothness of the numerical solution based on that stencil. WENO improves upon ENO in robustness, better smoothness of fluxes, better steady state convergence, better provable convergence properties, and more efficiency. For more details of ENO and WENO schemes, we refer to the lecture notes (106; 107).

WENO schemes have been widely used in applications. Some of the examples include dynamical response of a stellar atmosphere to pressure perturbations (26); shock vortex interactions and other gas dynamics problems (41; 42); incompressible flow problems (127); Hamilton-Jacobi equations (53); magneto-hydrodynamics (55); underwater blast-wave focusing (61); the composite schemes and shallow water equations (62; 63), real gas computations (78).

WENO method of cosmological hydrodynamic simulation

Though the universe seems to be dominated by the dark sides of both matter and energy (118), the observed luminous universe has been existing in the form of baryonic matter, whose mass density, constrained by the primordial nucleosynthesis (120), only occupies a small amount of the total density. To account for the observational features revealed by the baryonic matter, i.e., X-ray emitting gas in galaxies and clusters (79), intergalactic medium inferred from Lyα forest (91), X-ray background radiation (37) and distorted spectrum of the cosmic background radiation due to the Sunyaev-Zeldovich effect (129; 84), it would be necessary to incorporate the hydrodynamics into cosmological investigations. This motivation has stimulated great efforts to apply a variety of gas dynamics algorithms to cosmological simulations. For a general review of the state-of-the-art on this topic in non-relativistic and relativistic cases, we refer to, e.g. (7; 35; 71).

Although more than 10 cosmological hydrodynamical simulation codes have been proposed, there is still a need to develop codes based on new algorithms, with the objective of trying to obtain better performance for certain specific cosmological applications (it is probably impossible to have a code which performs better than others in all cosmological applications). In 1999, 11 codes were compared for their cluster simulation (33). The conclusion is that for thermal properties of clusters, such as entropy, X–ray luminosity etc., the results given by different codes are largely scattered. Therefore, new codes have been continuously proposed in recent years, e.g. (112; 113). The difficulty of the cosmological hydrodynamical simulation is due to the high non-linearity of gravitational Cosmological hydrodynamic flow poses more challenges than the typical hydrodynamic simulation without self-gravity. A significant feature is the extremely supersonic motion around the density peaks developed by gravitational instability, which leads to strong shock discontinuities within complex smooth structures. It would therefore be advantageous for the high order WENO schemes to be applied here, due to their capability to resolve both strong shocks and complicated smooth flow structures accurately at the same time. Qualified cosmological hydrodynamical simulation code should probably be able to pass two basic tests: 1. the Sedov-Taylor similarity solution or Bertschinger's similarity solution; and 2. the Zeldovich pancake solution. The results of these two tests for 13 codes are listed in the table below:

method	Sedov Blast wave	Zeldovich pancake
1. Bryan (13)	N/A	pass
2. Cen (Ryu) (100)	N/A	pass
3. Couchman (24)	fail	N/A
4. Evrard (27)	N/A	N/A
5. Gnedin (38)	N/A	N/A
6. Jenkins (87)	N/A	N/A
7. Navarro (81)	N/A	N/A
8. Owen (85)	pass	N/A
9. Pen (88)	pass	pass
10. Steinmetz (111)	N/A	N/A
11. Warren (121)	N/A	N/A
12. Springel (113)	pass	N/A
13. WENO	pass	pass

where “pass” or “fail” refers to the result when the relevant test has been performed and reported, and “N/A” refers to the situation where there is no report on whether the test has been performed.

Among the listed codes, the Lagrangian approach generally is based on the smoothed particle hydrodynamic (SPH) algorithm. A main challenge to the smoothed particle hydrodynamic (SPH) algorithm is the handling of shocks or discontinuities, because the nature of SPH is to smooth the fields considered (11; 83). For the Eulerian approach in numerical cosmology, the better codes are based on the high resolution shock capturing total-variation diminishing (TVD) scheme (Harten (45)) and piecewise parabolic method (PPM) (Collella and Woodward (23)). Both schemes start from the integral form of conservation laws of Euler equations and compute the flux vector based on cell averages (finite volume scheme). The TVD scheme modifies the flux using an approximate solution of the Riemann problem with corrections added to ensure that there are no postshock oscillations. While in the PPM scheme, the Riemann problem is solved accurately using a quadratic interpolation of the cell-average densities that is constrained to minimize postshock oscillations.

Recently, we have performed quantitative study of applying high order WENO schemes for solving physical problems governed by high Reynolds number Navier-Stokes equations (105; 132), and have obtained the conclusion that high order WENO schemes are more CPU time efficient in achieving the same level of accuracy or resolution than lower order schemes for problems containing both strong shocks and complicated smooth flow structures. This indicates that the WENO scheme has the potential to present a significant improvement on the cosmological hydrodynamical simulation, especially in considering the accuracy and better efficiency in the application of cosmological problems.

The WENO scheme has been demonstrated to be a good tool for simulating compressible turbulent flows with shocks in the recent study(72; 116; 124; 125). We will develop and adapt the WENO scheme to study the turbulence of cosmic fluid.

AMR algorithm of cosmic fluid

The WENO scheme takes a nonlinear adaptive procedure used to automatically choose the locally smoothest stencil, and hence avoiding crossing discontinuities in the interpolation procedure as much as possible. This algorithm provides a accurate way to capture shocks, especially when shock is curved. On the other hand, the smoothed particle hydrodynamic (SPH) algorithm may not be easy to handle shocks or discontinuities well because the nature of SPH is to smooth the fields (11; 83).

Nevertheless, to further study turbulence of the cosmic fluid, the WENO algorithm needs to be developed in considering that the Mach number of the cosmic fluid is very high, and the turbulence would be supersonic. In this case, the evolution of the turbulence is sensitive to the heating of shocks, as it can substantially change the sound speed. On the other hand, the thermal energy is less than the kinetic energy by several orders. This feature may cause errors in calculating the temperature and entropy changes because they are determined by the difference of the thermal energy on two sides of shock.

An effective way to improve the WENO algorithm for cosmic fluid is to use the adaptive mesh refinement (AMR), which is based on the finite-volume approach with a hierarchy of grid patches of different resolution (82). The idea is to represent relatively smooth flow regions on coarse grids, while the regions with steep gradients are computed with grids of higher resolution. Thus, we may have a better numerical result of the identification of shocks. It has been noted that the turbulence may lead to spatial correlation on the large scale, such as large scale vortical mode. This might pose a challenge to the AMR of turbulent fluid. We plan to develop a fully conservative and high order finite difference WENO scheme within the AMR framework for cosmological simulation, which should save computational cost significantly over finite volume WENO schemes. This point will be elaborated more in Section 2.4.4.

3.1.2 Algorithm with Galerkin discretization

The need of Galerkin discretization

In cosmological and astrophysical study, sometimes we need to find maps $g_i(\mathbf{r})$ from observed maps $f_j(\mathbf{r})$, with the theoretical relationships between

$g(\mathbf{r})$ are $f(\mathbf{r})$ being differential, such as

$$Of = g, \tag{3.1.1}$$

where O is a linear continuous operator, like Laplace or derivative, and f and g are functions in space $\mathbf{r}$. However, observed maps generally are pixelized. That is, we do not know $f(\mathbf{r})$, but only $\tilde{f}_i$, which is given in N pixels (cells) of a finite space. That is, we have

$$f(\mathbf{r}) = \sum_{i=1}^{N} \tilde{f}_i w_i(\mathbf{r}). \tag{3.1.2}$$

where the function $w_i(\mathbf{r})$ is the binning function of pixel i.

The operator O, such as a derivative ∂_x, is a continuous linear operator to map functions defined in a Hilbert space, while the maps $\tilde{f}_j$ are defined in the the N-dimensional space spanned by bases $[v_1(\mathbf{r}), ...v_N(\mathbf{r})]$. Therefore, we should approximate the derivative operator from a mapping between functions defined in a Hilbert space, to a mapping in a subspace spanned by v_i. This is a Galerkin problem.

With eqs.(3.1.1,3.1.2), we have $g_i \equiv \langle g, v_i \rangle = \sum_{k=1}^{N} \langle O w_k, v_i \rangle \tilde{f}_k$, where the matrix $O_{ik} \equiv \langle O w_k, v_i \rangle$ is a discretization of operator O from the space $\mathbf{r}$ to a finite-dimensional subspace $[v_i]$. A Galerkin discretization requires the following equation to be held

$$\langle g, v_i \rangle = \langle Of, v_i \rangle. \tag{3.1.3}$$

That is, for any functions of the N-dimensional space spanned by the bases $v_i(\mathbf{r})$, equation (3.1.3) should be held.

The discretization of operator O actually is inevitable for all algorithms on the data $\tilde{f}_i$. Any algorithm on eq.(3.1.1) with the base v_i in the spatial domain will reach $g_i = \sum_{k=1}^{N} \langle O w_k, v_i \rangle \tilde{f}_k$, regardless of whether eq.(3.1.3) holds. The condition of Galerkin discretization, eq.(3.1.3), gives the best discretization (60). To properly handle the observed maps, we developed efficient algorithms for solving differential equations incorporating the Galerkin discretization.

Galerkin algorithm of E/B mode separation

The perturbations of space-time metric at inflation can cause both scalar and tensor components of the primordial perturbations. The tensor component can be probed by the Stokes parameter of the linear polarization of CMB. A tensor field generally contains electric-like E-mode and magnetic-like B-mode. In linear regime, the perturbed density field initially can only yield the E-mode, but not the B-mode. On the other hand, the B mode can be produced by gravitational waves. Therefore, to extract B-mode information from CMB polarization maps is crucial to verify the existence of gravitational wave background produced at inflationary epoch. Moreover, gravitational lensing of clusters and hot electron scattering of reionization would be able to yield both E and B-modes. Therefore, an effective algorithm of the decomposition of E and B modes from observed maps of Stokes parameters is required.

The challenge of E/B mode separation comes from the differential relation between the maps of observed Stokes parameters and the E, B modes. The observed maps actually are discrete, noisy, and in finite area. The derivative operator acting on the discrete and noisy maps generally leads to large numerical errors. It leads to a leakage of E power to B power. Therefore, it is difficult to extract the information of B mode from observed polarization maps of the CMB if the power of B mode is much less than the E mode.

A Galerkin discretization of the differential operator would be helpful. With proper Galerkin discretization, the errors can be reduced, especially, it does not produce false correlations. For instance, with wavelet-Galerkin discretization, the discretized differential operator is represented by a matrix, which is exactly diagonal in the scale space, and narrowly banded in the spatial space. We developed wavelet-Galerkin method to draw the B mode maps from the observed CMB polarization maps.

3.2 Turbulence of cosmic baryonic fluid

3.2.1 Turbulent cosmic fluid

Although the IGM is a Navier-Stokes fluid, and dynamically governed by the gravity of dark matter, it has been recognized that the growth modes of cosmic baryon gas in nonlinear regime is like Burgers fluid. The theory of Zeldovich's pancake and the succeed adhesion model first indicated that the non-linear evolution of the gravitational clustering of cosmic matter can be sketched by a Burgers' equation (43). Considering that the cosmic matter is dissipative, the dynamical equation of the velocity field of the large scale structure is found essentially to be a variant of the random-force-driven Burgers equation or the KPZ equation (6). Later, with a two-component (dark matter and baryon gas) generalization of the adhesion model, it was also found that the velocity potential of the baryon gas is described by the Burgers equation driven by the dark matter gravitational potential (6; 56; 74). Consequently, when the Reynolds number is large (in nonlinear regime, IGM generally is in this case), the Burgers turbulence will be developed.

We have showed that the velocity field of cosmic baryon fluid in the nonlinear regime is intermittent(58). In the scale range from the Jeans length to about 16 h^{-1} Mpc, this field can be extremely well described by the She-Lévěque's scaling formula. The baryon fluid also possesses the features:

(1) for volume weight statistics, the dissipative structures are dominated by sheets, and (2) the relation between the intensities of fluctuations is hierarchical. These results imply that the evolution of highly evolved cosmic baryon fluid is similar to a fully developed turbulence(47; 101).

3.2.2 Log-Poisson hierarchy of cosmic baryon fluid

In the scale-free range, the non-Gaussian features of the mass density field of cosmic baryon fluid can be well described by a log-Poisson hierarchical cascade, which yields the She-Lévěque's scaling. All the predictions given by the log-Poisson RMP model, including the hierarchical relation, the order dependence of the intermittent exponent, the moments, and the scale-scale correlation, are found to be in good agreement with the statistical results from 2nd to, at least, 12th orders of the samples of hydrodynamic simulation(67). With hydrodynamic simulation samples of the concordance ΛCDM universe, we further show that the mass density field of neutral hydrogen, is also well described by the log-Poisson hierarchy.

We then investigate the field of Lyα transmitted flux of QSO absorption spectrum (30; 52; 86; 130). Due to redshift distortion, Lyα transmitted flux fluctuations are no longer to show all features of the log-Poisson hierarchy. However, some non-Gaussian features predicted by the log-Poisson hierarchy are not affected by the redshift distortion. We test these predictions with the high resolution and high S/N data of quasars Lyα absorption spectra. All statistical results given by real data, including β-hierarchy, high order moments and scale-scale correlation, are found to be well consistent with the log-Poisson hierarchy. We compare the log-Poisson hierarchy with the popular log-normal model of the Lyα transmitted flux. The non-Gaussianity given by later is too strong at high orders, while the log-Poisson hierarchy is in agreement with observed data.

This result is also useful to understand both the dynamical and thermal properties of cosmic hydrogen gas. A comparison of the absorption features of HeII and HI of HE2347-4342 indicates that the absorption lines probably are turbulent-broadening(133).

3.2.3 Vorticity of cosmic fluid

The vorticity can only be produced by nonlinear processes, and therefore, the field of vorticity is fundamentally important to describe the evolved cosmic fluid. The dynamical equation of vorticity is free from gravity. It is very helpful to study the statistical and dynamical decoupling of cosmic baryon matter from the underlying dark matter. Moreover, the relation between the vorticity and velocity is nonlocal, and hence, vorticity is more effective to "feel" clustering happening at a distant position.

High order accurate solutions of hydrodynamics allows us to reasonably estimate the turbulence pressure, which is an important factor to prevent the gravitational collapsing of the cosmic fluid (20; 9). This approach can directly be applied to study 1) the baryon fractions in the halos of collapsed objects generally are lower than cosmic fraction of baryons (57; 5; 128), and 2.) how the mass center of baryon matter around a hole is offsetting the mass center of dark matter (12).

The kSZ effect is given by a line integral over the momentum density field of the IGM along a line of sight. The integral should be very small if the velocity field is irrotational, or curl-free (119; 131). However, it is sensitive to the vorticity of the IGM velocity field. Therefore, the kSZ would be an effective tool to probe the turbulence of IGM. Since turbulent IGM yields special non-Gaussianity, we will study the high order behavior of the kSZ map.

Turbulence enhances the transfer of kinetic energy into thermal energy. This process actually is a dissipation of the kinetic energy of vortical structures on large scales. It will yield entropy production of turbulent cosmic fluid. Considering vorticity of turbulent fluid probably would be an origin of magnetic field in cosmic fluid (25; 100), magnetic field would also join the dissipation. We will study these problems with the WENO algorithm.

We should emphasize again that the WENO scheme has been demonstrated to be a good tool for simulating compressible turbulent flows with shocks in the recent study (72; 116; 124; 125). We will develop and adapt the WENO scheme for our specific models here to study the turbulence effects.

3.2.4 Scaling relations of clusters

As a result of the turbulence, the relations among various physical quantities of cosmic baryon fluid should be scale invariant, if the physical quantities are measured in cells on scales larger than the dissipation scale. We examine this property with the relation between the Compton parameter of the thermal Sunyaev-Zeldovich effect, $y(r)$, and X-ray luminosity, $Lx(r)$, where r being the scale of regions in which y and Lx are measured. According to the self-similar hierarchical scenario of nonlinear evolution, one should expect that 1.) in the $y(r) - Lx(r)$ relation, $y(r) = 10A(r)[Lx(r)]^{\alpha(r)}$, the coefficients $A(r)$ and $\alpha(r)$ are scale-invariant; 2.) The relation $y(r) = 10A(r)[Lx(r)]^{\alpha(r)}$ given by cells containing collapsed objects is also available for cells without collapsed objects, only if r is larger than the dissipation scale. These two predictions are well established with a scale decomposition analysis of observed data, and a comparison of observed $y(r) - Lx(r)$ relation with hydrodynamic simulation samples.

3.3 Time-dependent behavior of resonant photo transfer

Lyα photons have been widely applied to study cosmological problems in the redshift range from 2 to 6. The time-dependent problem of Lyα photon transfer becomes serious at high redshifts, as the time scales of high redshift sources generally are short. Since early 1970s, many solutions on the radiative transfer of Lyα photons in optically thick medium have been done analytically and numerically. However, the time-dependent analytical solutions are available only for the cases that the distributions of sources and IGM are homogeneous (32). Right now, there are very few numerical solvers of the time-dependent problem of radiative transfer equation with resonant scattering. The newest time-dependent solver (75) still cannot pass the test of analytical solutions (32). The Monte Carlo simulation method is capable to study the time-dependent problems. However, only the time scale of the photon escaping with a "single longest excursion" is estimated by the Monte Carlo method (2; 10). It has not yet show even the time evolution of the local thermal equilibrium of Wouthuysen-Field (W-F) effect.

On the other hand, the WENO solver can very well pass these tests (96). We have successfully to solve the radiative transfer by using the WENO scheme for both the one dimensional (one space dimension, two phase dimension plus time) and two dimensional (two space dimension, three phase dimension plus time) situations (14; 15; 16). The WENO code developed in these references perform significantly better than other solvers.

An eventual development of a WENO code for radiative transfer has been done in the time-dependent problems of resonant photo transfer. It reveals many interesting features of the time evolution of resonant photons Lyα in optical thick medium, including 1) the time-scale of the formation of the W-F effect; 2) the light-curve of a flash sources surrounded by optically thick halo; 3.) the deviation of two-peak structure of escaped Lyα from the analytical solution of the Fokker-Planck approximation (28; 96; 97; 98; 99).

3.3.1 The growth of ionized and heated regions

Ionized and heated regions are important problem of early universe. There are also two approaches on the evolution of radiation field. The first is to describe the reionization with a rate equation of ionizing photons, i.e. the conservation equation of the ionizing photon number(102; 70; 76). The second approach is based on static radiative transfer, i.e. to drop the time derivative term of the radiative transfer equation. It is equal to assume the speed of light is infinite(92; 1; 22; 40; 110; 80; 93; 18; 73; 103; 95; 51; 115; 122).

Our works revealed that all the above-mentioned solutions are shortage in the following three aspects. 1. Their approximations would be reasonable only if the retardation effect is negligible. The retardation effect is not trivial. The time- and space-dependencies of the ionized region are substantially affected by the retardation(123; 104; 89; 90). This problem is more serious at high redshift, as the time scale of the retardation is comparable with the age of the universe (90). 2. The effects related to the time-dependence of photon's frequency spectrum are omitted. However, For instance, to calculate the heating or the temperature profile if IGM around UV photon sources, the evolution of the photon distribution function in phase space is essential. 3. The dynamical behavior of cosmic baryon fluid is treated separately with the evolution of radiation. This approximation is reasonable only it the time scale of photon's retardation is much less than that of cosmic baryon fluid. However, it will not be so at high redshift epoch. It would also therefore be advantageous for the WENO algorithm, which can be the solver of evolution of baryon fluid in density and velocity spaces, and radiations in phase space.

3.3.2 Wouthuysen-Field effect

The radiative transfer of Lyα photons and other resonant photons is crucial to study the physics of reionization epoch of the universe.

First, the 21 cm emission and absorption from gaseous halos around the first generation of stars depend on the Wouthuysen-Field (W-F) coupling, which relates the spin temperature with the kinetic temperature of hydrogen gas via the resonant scattering between Lyα photons and neutral hydrogen. Therefore, the transfer of Lyα photons in both spatial and frequency spaces are essential to predict the 21 cm signal from the halos. The time-dependence of the transfer is especially important considering that the lifetime of first stars generally is short. There are very few solver of the time-dependent radiative transfer equation with resonant scattering(75). These solvers generally cannot pass the test of analytical solutions (32). On the other hand, the WENO solver is found to be very well pass these tests.

Second, the profile of red damping wing of high redshift Gamma Ray Burst (GRB) spectrum is determined by the distribution of the neutral hydrogen. Therefore, the red damping wing of high redshift GRB would be very useful to probe the neutral hydrogen at reionization. However, the profile generally is calculated by a Gunn-Peterson absorption without considering the resonant scattering (94). If the red damping wing is mainly due to the absorption and re-emission of neutral hydrogen located at the position close to the GRB, the resonant scattering should be considered. That is, to extract information of the reionization epoch, we should find the profile of red damping wing by solving the radiative transfer equation with resonant scattering. Some profiles have been given by our solution of the radiative transfer with resoant scattering.

Third, the absorption spectra of quasars at redshift $z > 5$ consist of complete absorption troughs (Gunn-Peterson troughs) separated by tiny regions, which are Gunn-Peterson transparent(4; 29). The transparent might have two origins: a.) the vestige of ionized patch around first generation stars and b.) low mass density areas of hydrogen gas at early universes. Obviously, to discriminate the two models, it is necessary to identify the features of leaks given by the two models. Since the WENO schemes are able to handle both of the hydrodynamics of baryon fluid and radiative transform, it is able to do the simulation of the evolution of ionized patch, i.e. the evolutions of the profiles of HI, HII in the physical space, and radiative distribution in the phase space.

3.4 Physics of reionization epoch

The onset of the first generation star formation marks the time at which the Universe emerged from the so-called "Dark Ages" and is referred to as the epoch of reionization. The physics of the epoch of reionization is one of the most important topics in cosmology. It has attracted many theoretical investigations after the SDSS and WMAP showing some evidences of the reionization (29; 59). Hydrodynamics and radiative transfer are essential to study the formation of first stars and the reionization of neutral hydrogen clouds. How does the gas evolve from the state of primordial baryons to clustered clouds suitable to form the first generation stars? What are the major sources of heating neutral gas? How long did the history last from the first star formation to a full reionization? Can the clustering of primeval hydrogen gas be described by a similar mapping of the collapsing of dark matter? What are the statistical features of the spatial distributions of gaseous temperature and density of neutral and ionized hydrogen during the first star formation? What are the statistical features of the redshifted 21 cm emission of the neutral hydrogen? Preliminary theoretical investigations done by different groups, including ours, have shown that these problems are complicated (17; 44; 126; 49; 8; 64; 48). Although simeanalytical models can provide interesting results, but they are far from enough. A precise hydrodynamic simulation is necessary.

3.4.1 21 cm emission and absorption at early universe

Cosmic 21 cm signal is due to the decoupling of the spin temperature of neutral hydrogen atoms from the temperature of CMB. Therefore, the detection of redshifted 21 cm signals from the early universe is attracting many attentions in the study of cosmology(36; 48). The 21 cm signals from individual UV ionizing sources in the reionization epoch may provide a direct identification of the ionized patches of the reionization(117; 21; 19; 66). All these calculation rely on the so-called Wouthuysen-Field mechanism, which leads to the color temperature of the radiation spectrum near Lyα frequency to be equal to the kinetic temperature of the baryonic gas if the resonant scattering of Lyα photons by neutral hydrogen is effective. If there are enough Lyα photons, the Wouthuysen-Field mechanism works well.

There are more questions about the 21 cm signal in the early universe. For instance, whether there are correlation between the Lyα leaks and 21 cm absorption, and whether died sources will left a 21 cm absorption region. These problems are related to physics of non-thermal equilibrium. It require a detail calculation of the evolution of the Lyα photons in phase space caused by the resonant scattering. Since the resonant scattering will lead to rapid change of photon distribution function in frequency space, the WENO method would be advantaged.

3.4.2 Leaking of high redshift quasar's absorption spectrum

The absorption spectra of quasars at redshift $z > 5$ consist of complete absorption troughs (Gunn-Peterson troughs) separated by tiny regions, which are Gunn-Peterson transparent and lead to Lyα photon leaking(4; 29). The nature of the leaking is crucial to understand the physics of reionization. According to commonly accepted scenario of reionization, at early stage, only isolated patches around ionizing sources are highly ionized. The subsequent growing and overlapping of the ionizing patches lead to a uniform ionizing background and the end of reionization(22; 110; 39; 76). The ionization fraction of the IGM and the ionizing radiation underwent an evolution from highly non-uniform patches to a quasi-homogeneous field. Before the patch-to-uniform transition, only ionized patches would be transparent to Lyα photons. After the transition, the low density voids will also be Gunn-Peterson transparent. Therefore, Lyα leaks might have two origins: 1. the vestige of ionized patch around first generation stars and 2. low mass density areas of hydrogen gas at early universes. We find the model 2 is consistent with all the statistical features of Lyα leaks at redshift from 5 to 6.

Lyα leaks originate from low mass density areas (voids) is important for cosmology, because the statistics of Lyα leaks actually is the statistics of voids formed in the early universe. The size (or width) and its distribution of Lyα leaks would be similar to the mass function of galactic clusters. The zise (width) distribution of voids would be able to yield effective constrain on cosmological parameters.

It has been pointed out that the transition of the ionization state of cosmic hydrogen is similar to similar to a phase transition, and mean optical depth plays the role as order parameter(8; 65). Therefore, like phase transition in general the correlation length of the Lyα underwent a dramatic evolution during the phase transition.

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4 Publications (2005 - 2009)

Refereed journals

1. Distributions of baryon fraction on large scales in the universe, P. He, L.L. Feng and L.Z. Fang, *Astrophys. J.*, **623**, 601, (2005)

   The nonlinear evolution of a system consisting of baryons and dark matter is generally characterized by strong shocks and discontinuities. The baryons slow down significantly at postshock areas of gravitational strong shocks, which can occur in high overdense as well as low overdense regions. Consequently, the baryon fraction would be nonuniform on large scales. We studied these phenomena with simulation samples produced by the WENO hybrid cosmological hydrodynamic/N-body code. We find that the baryon fraction in high mass density regions is lower on average than the cosmic baryon fraction, and many baryons accumulate in the regions with moderate mass density to form a high baryon fraction phase (HBFP). In dense regions with rho¿100, which are the possible hosts for galaxy clusters, the baryon fraction can be lower than the cosmic baryon fraction by about 10%–20% at z 0. Our simulation samples show that about 3% of the cosmic baryon budget was hidden in the HBFP at redshift z=3, while this percentage increases to about 14% at the present day. The gas in the HBFP cannot be detected either by Ly-alpha forests of QSO absorption spectra or by soft X-ray background. That is, the HBFP would be missed in the baryon budget given by current observations.

2. The velocity field of baryonic gas in the universe, B. Kim, P.He, J. Pando, L.L. Feng and L. Z. Fang, *Astrophys. J.*, **625**, 599, (2005)

   The dynamic evolution of the baryonic intergalactic medium (IGM) caused by the underlying dark matter gravity is governed by the Navier-Stokes equations in which many cooling and heating processes are involved. However, it has long been recognized that the growth mode dynamics of cosmic matter clustering can be sketched by a random force driven Burgers' equation if cooling and heating are ignored. Just how well the dynamics of the IGM can be described as a Burgers fluid has not been fully investigated probably because cooling and heating are essential for a detailed understanding of the IGM. Using IGM samples produced by a cosmological hydrodynamic simulation in which heating and cooling processes are properly accounted for, we show that the IGM velocity field in the nonlinear regime shows the features of a Burgers fluid, that is, when the Reynolds number is high, the velocity field consists of an ensemble of shocks. Consequently, (1) the IGM velocity v is generally smaller than that of dark matter; (2) for the smoothed field, the IGM velocity shows tight correlation with dark matter given by $v \simeq sv_{dm}$, with $s < 1$, such that the lower the redshift, the smaller s; (3) the velocity PDFs are asymmetric between acceleration and deceleration events; (4) the PDF of velocity difference $\Delta v = v(x+r) - v(x)$ satisfies the scaling relation for a Burgers fluid, i.e., $P(\Delta v) = (1r^y)F(\Delta v/r^y)$. We find the scaling function and parameters for the IGM which are applicable to the entire scale range of the samples (0.26 - 8 h^{-1} Mpc). These properties show that the similarity mapping between the IGM and dark matter is violated on scales much larger than the Jeans length of the IGM.

3. A parameter-free statistical measurement of halos with power spectra, P. He, L.L. Feng and L.Z. Fang, *Astrophys. J.*, **628**, 14, (2005)

   We show that, in the halo model of large-scale structure formation, the difference between the Fourier and the DWT (discrete wavelet transform) power spectra provides a statistical measurement of the halos. This statistical quantity is free from parameters related to the shape of the mass profile and the identification scheme of halos. That is, the statistical measurement is invariant in the sense that models with reasonably defined and selected parameters of the halo models should yield the same difference of the Fourier and DWT spectra. This feature is useful to extract ensemble averaged properties of halos, which cannot be obtained with the identification of individual halo. To demonstrate this point, we show with WIGEON hydrodynamical simulation samples that the spectrum difference provides a quantitative measurement of the discrepancy of the distribution of baryonic gas from that of the underlying dark matter field within halos. We also show that the mass density profile of halos in physical space can be reconstructed with this statistical measurement. This profile essentially is the average over an ensemble of halos, including well virialized halos as well as halos with significant internal substructures. Moreover, this reconstruction is sensitive to the tail of the mass density profile. We showed that the profile with $1/r^3$ tail gives very different result from that of $1/r^2$. Other possible applications of this method are discussed as well.

4. Power spectrum and intermittency of Lymanα transmitted flux of QSO HE2347-4342, P. Jamkhedkar, L.L. Feng, W. Zheng and L.Z. Fang, *Astrophys. J.*, **633**, 52, (2005)

   We have studied the power spectrum and the intermittent behavior of the fluctuations in the transmitted flux of HE2347-4342 Lyα absorption in order to investigate if there is any discrepancy between the LCDM model with parameters given by the WMAP and observations on small scales. If the non-Gaussianity of cosmic mass field is assumed to come only from halos with an universal mass profile of the LCDM model, the non-Gaussian behavior of mass field would be effectively measured by its intermittency, because intermittency is a basic statistical feature of the cuspy structures. We have shown that the Lyα transmitted flux field of HE2347-4342 is significantly intermittent on small scales. With the hydrodynamic simulation, we demonstrate that the LCDM model is successful in explaining the power spectrum and intermittency of Lyα transmitted flux. Using statistics ranging from the second to eighth order, we find no discrepancy between the LCDM model and the observed transmitted flux field, and no evidence to support the necessity of reducing the power of density perturbations relative to the standard LCDM model up to comoving scales as small as about 0.08h^{-1}Mpc. Moreover, our simulation samples show that the intermittent exponent of the Lyα transmitted flux field is probably scale-dependent. This result is different from the prediction of universal mass profile with a constant index of the central cusp. The scale-dependence of the intermittent exponent indicates that the distribution of baryonic gas is decoupled from the underlying dark matter.

5. Low-redshift cosmic baryon fluid on large scales and She-Levueque's universal scaling, P. He, J.R. Liu, L.L. Feng, C.W. Shu and L.Z. Fang, *Phys. Rev. Lett.*, **96**, 051302, (2006)

   We investigate the statistical properties of cosmic baryon fluid in the nonlinear regime, which is crucial for understanding the large-scale structure formation of the universe. With the hydrodynamic simulation sample of the Universe in the cold dark matter model with a cosmological constant, we show that the intermittency of the velocity field of cosmic baryon fluid at r edshift z=0 in the scale range from the Jeans length to about 16 Mpc/h can be extremely well described by She-Leveque's universal scaling formula. The baryon fluid also possesses the following features: (1) for volume weight statistics, the dissipative structures are dominated by sheets, and (2) the relation between the intensities of fluctuations is hierarchical. These results imply that the evolution of highly evolved cosmic baryon fluid is similar to a fully developed turbulence.

6. X-ray emission of baryonic gas in the universe: luminosity-temperature relationship and soft band background, T.J. Zhang, J.R. Liu, L.L. Feng, P. He and L.Z. Fang, *Astrophys. J.*, **642**, 625, (2006)

   We study the X-ray emission of baryon fluid in the universe using the WIGEON cosmological hydrodynamic simulations. It has been revealed that cosmic baryon fluid in the nonlinear regime behaves like Burgers turbulence, i.e. the fluid field consists of shocks. Like turbulence in incompressible fluid, the Burgers turbulence plays an important role in converting the kinetic energy of the fluid to thermal energy and heats the gas. We show that the simulation sample of the ΛCDM model without adding extra heating sources can fit well the observed distributions of X-ray luminosity versus temperature (L_x vs. T) of galaxy groups and is also consistent with the distributions of X-ray luminosity versus velocity dispersion (L_x vs. σ). Because the baryonic gas is multiphase, the $L_x - T$ and $L_x - \sigma$ distributions are significantly scattered. If we describe the relationships by power laws $L_x \propto T^{\alpha_{LT}}$ and $L_x \propto \sigma^{\alpha_{LV}}$, we find $\alpha_{LT} > 2.5$ and $\alpha_{LV} > 2.1$. The X-ray background in the soft $0.5 - 2$ keV band emitted by the baryonic gas in the temperature range $10^5 < T < 10^7$ K has also been calculated. We show that of the total background, (1) no more than 2% comes from the region with temperature less than $10^{6.5}$ K, and (2) no more than 7% is from the region of dark matter with mass density $\rho_{\rm dm} < 50\bar{\rho}_{\rm dm}$. The region of $\rho_{\rm dm} > 50\bar{\rho}_{\rm dm}$ is generally clustered and discretely distributed. Therefore, almost all of the soft X-ray background comes from clustered sources, and the contribution from truly diffuse gas is probably negligible. This point agrees with current X-ray observations.

7. Cross-correlation between WMAP and 2MASS: non-Gaussianity induced by SZ effect, L. Cao, Y.Q. Chu and L.Z. Fang, *Mon. Not. R. Astr. Soc.*, **369**, 645, (2006)

   We study the SZ-effect-induced non-Gaussianity in the cosmic microwave background (CMB) fluctuation maps. If a CMB map is contaminated by the SZ effect of galaxies or galaxy clusters, the CMB maps should have similar non-Gaussian features as the galaxy and cluster fields. Using the WMAP data and 2MASS galaxy catalog we show that the non-Gaussianity of the 2MASS galaxies is imprinted on WMAP maps. The signature of non-Gaussianity can be seen with the 4^{th} order cross correlation between the wavelet variables of the WMAP maps and 2MASS clusters. The intensity of the 4^{th} order non-Gaussian features is found to be consistent with the contamination of the SZ effect of 2MASS galaxies. We also show that this non-Gaussianity can not be seen by the high order auto-correlation of the WMAP. This is because the SZ signals in the auto-correlations of the WMAP data generally is weaker than the WMAP-2MASS cross correlations by a factor f^2, which is the ratio between the powers of SZ effect map and the CMB fluctuations on the scale considered. Therefore, the ratio of high order auto-correlations of CMB maps to cross-correlations of the CMB maps and galaxy field would be effective to constrain the powers of SZ effect on various scales.

8. Is the cosmic UV background fluctuating at redshift $z \simeq 6$? J.R. Liu, H.G Bi, L.L. Feng and L.Z. Fang, *Astrophys. J. Lett.* **645**, L1, (2006)

   We study the Gunn-Peterson effect of the photo-ionized intergalactic medium(IGM) in the redshift range 5¡ z ¡6.4 using semi-analytic simulations based on the lognormal model. Assuming a rapidly evolved and spatially uniform ionizing background, the simulation can produce all the observed abnormal statistical features near redshift z 6. They include: 1) rapidly increase of absorption depths; 2) large scatter in the optical depths; 3) long-tailed distributions of transmitted flux and 4) long dark gaps in spectra. These abnormal features are mainly due to rare events, which correspond to the long-tailed probability distribution of the IGM density field, and therefore, they may not imply significantly spatial fluctuations in the UV ionizing background at z 6.

9. A unified fitting of HI and HeII Lyα transmitted flux of QSO HE2347 with ΛCDM hydrodynamic simulations, J.R. Liu, P. Jamkhedkar, W. Zheng, L.L. Feng, and L. Z. Fang, *Astrophys. J.*, **645**, 861, (2006)

   Using cosmological hydrodynamic simulations of the LCDM model, we present a comparison between the simulation sample and real data sample of HI and HeII Lyα transmitted flux in the absorption spectra of the QSO HE2347-4342. The LCDM model is successful in simultaneously explaining the statistical features of both HI and HeII Lyα transmitted flux. It includes: 1.) the power spectra of the transmitted flux of HI and HeII can be well fitted on all scales ¿ 0.28h^{-1} Mpc for H, and ¿ 1.1h^{-1} Mpc for He; 2.) the Doppler parameters of absorption features of HeII and HI are found to be turbulent-broadening; 3.) the ratio of HeII to HI optical depths are substantially scattered, due to the significant effect of noise. A large part of the η-scatter is due to the noise in the HeII flux. However, the real data contain more low-η events than simulation sample. This discrepancy may indicate that the mechanism leading extra fluctuations upon the simulation data, such as a fluctuating UV radiation background, is needed. Yet, models of these extra fluctuations should satisfy the constraints: 1.) if the fluctuations are Gaussian, they should be limited by the power spectra of observed HI and HeII flux; 2.) if the fluctuations are non-Gaussian, they should be limited by the observed non-Gaussian features of the HI and HeII flux.

10. A WENO algorithm for the radiative transfer and ionized sphere at reionization, J.-M. Qiu, C.-W. Shu, L.-L. Feng and L. Z. Fang, *New Astronomy*, **12**, 1, (2006)

    We show that the algorithm based on the weighted essentially nonoscillatory (WENO) scheme with anti-diffusive flux corrections can be used as a solver of the radiative transfer equations. This algorithm is highly stable and robust for solving problems with both discontinuities and smooth solution structures. We test this code with the ionized sphere around point sources. It shows that the WENO scheme can reveal the discontinuity of the radiative or ionizing fronts as well as the evolution of photon frequency spectrum with high accuracy on coarse meshes and for a very wide parameter space. This method would be useful to study the details of the ionized patch given by individual source in the epoch of reionization. We demonstrate this method by calculating the evolution of the ionized sphere around point sources in physical and frequency spaces. It shows that the profile of the fraction of neutral hydrogen and the ionized radius are sensitively dependent on the intensity of the source.

11. A WENO algorithm of the temperature and ionization profiles around a point source, J.M. Qiu, L. L. Feng, C.W. Shu and L. Z. Fang, *New Astronomy*, **12**, 398, (2007)

    We develop a numerical solver for radiative transfer problems based on the weighted essentially nonoscillatory (WENO) scheme modified with anti-diffusive flux corrections, in order to solve the temperature and ionization profiles around a point source of photons in the reionization epoch. Algorithms for such simulation must be able to handle the following two features: 1. the sharp profiles of ionization and temperature at the ionizing front (I-front) and the heating front (T-front), and 2. the fraction of neutral hydrogen within the ionized sphere is extremely small due to the stiffness of the rate equations of atom processes. The WENO scheme can properly handle these two features, as it has been shown to have high order of accuracy and good convergence in capturing discontinuities and complicated structures in fluid as well as to be significantly superior over piecewise smooth solutions containing discontinuities. With this algorithm, we show the time-dependence of the preheated shell around a UV photon source. In the first stage the I-front and T-front are coincident, and propagate with almost the speed of light. In later stage, when the frequency spectrum of UV photons is hardened, the speeds of propagation of the ionizing and heating fronts are both significantly less than the speed of light, and the heating front is always beyond the ionizing front. In the spherical shell between the I- and T-fronts, the IGM is heated, while atoms keep almost neutral. The time scale of the preheated shell evolution is dependent on the intensity of the photon source. We also find that the details of the pre-heated shell and the distribution of neutral hydrogen remained in the ionized sphere are actually sensitive to the parameters used. The WENO algorithm can provide stable and robust solutions to study these details.

12. Estimating power spectrum of Sunyaev-Zeldovich effect from the cross-correlation between WMAP and 2MASS, L. Cao, J.R. Liu, and L.Z. Fang, *Astrophys. J.*, **661**, 641, (2007)

    Abstract: We estimate the power spectrum of SZ(Sunyaev-Zel'dovich)-effect-induced temperature fluctuations on sub-degree scales by using the cross correlation between the three-year WMAP maps and 2MASS galaxy distribution. We produced the SZ effect maps by hydrodynamic simulation samples of the ΛCDM model, and show that the SZ effect temperature fluctuations are highly non-Gaussian. The PDF of the temperature fluctuations has a long tail. More than 70% power of the SZ effect temperature fluctuations attributes to top $\sim 1\%$ wavelet modes (long tail events). On the other hand, the CMB temperature fluctuations basically are Gaussian. Although the mean power of CMB temperature fluctuations on sub-degree scales is much higher than that of SZ effect map, the SZ effect temperature fluctuations associated with top 2MASS clusters is comparable to the power of CMB temperature fluctuations on the same scales. Thus, from noisy WMAP maps, one can have a proper estimation of the SZ effect power at the positions of the top 2MASS clusters. The power spectrum given by these top wavelet modes is useful to constrain the parameter of density fluctuations amplitude σ_8. We find that the power spectrum of these top wavelet modes of SZ effect on sub-degree scales basically is consistent with the simulation maps produced with $\sigma_8 = 0.84$. The simulation samples of $\sigma_8 = 0.74$ show, however, significant deviation from detected SZ power spectrum. It can be ruled out with confidence level 99% if all other cosmological parameters are the same as that given by the three-year WMAP results.

13. 21 cm signals from early ionizing sources, J. R. Liu, J.M. Qiu, L. L. Feng, C. W. Shu and L.Z. Fang *Astrophys. J.*, **663**, 1, (2007)

    We investigate the 21 cm signals from the UV ionizing sources in the reionization epoch. The formation and evolution of 21 cm emission and absorption regions depend essentially on the kinetics of photons in the physical and frequency spaces. To solve the radiative transfer equation, we use the WENO algorithm, which is effective to capture the sharp ionization profile and the cut-off at the front of light $(r = ct)$ and to handle the small fraction of neutral hydrogen and helium in the ionized sphere. We show that a spherical shell of 21 cm emission and absorption will develop around a point source once the speed of the ionization front (I-front) is significantly lower than the speed of light. The 21 cm shell extends from the I-front to the front of light; its inner part is the emission region and its outer part is the absorption region. The 21 cm emission region depends strongly on the intensity, frequency-spectrum and life-time of the UV ionizing source. For a source of short life-time, no 21 cm emission region can be formed if the source dies out before the I-front speed is significantly lower than the speed of light. Yet, a 21 cm absorption region can form and develop even after the emission of the source ceases.

14. Lyα Leaks in the Absorption Spectra of High Redshift QSOs, J.R Liu, H.G. Bi & L.Z. Fang, *Astrophys. J. Lett.*, **671**, L89, (2007)

    Spectra of high redshift QSOs show deep Gunn-Peterson absorptions on the blue sides of the Lya emissions lines. They can be decomposed into components called Lya leaks, defined to be emissive regions in complementary to otherwise zero-fluxed absorption gaps. Just like Lya absorption forests at low redshifts, Lya leaks are both easy to find in observations and containing rich sets of statistical properties that can be used to study the early evolution of the IGM. Among all properties of a leak profile, we investigate its equivalent width in this paper, since it is weakly affected by instrumental resolution and noise. Using 10 Keck QSO spectra at $z \sim 6$, we have measured the number density distribution function $n(W, z)$, defined to be the number of leaks per equivalent width W and per redshift z, in the redshift range $5.4 - 6.0$. These new observational statistics, in both the differential and cumulative forms, fit well to hydro numerical simulations of uniform ionizing background in the ΛCDM c osmology. In this model, Ly α leaks are mainly due to low density voids. It supports the early studies that the IGM at $z \simeq 6$ would still be in a highly ionized state with neutral hydrogen fraction $\simeq 10^{-4}$. Measurements of $n(W, z)$ at $z > 6$ would be effective to probe the reionization of the IGM.

15. A WENO algorithm for the growth of ionized regions at the reionization epoch, J. M. Qiu, C.W. Shu, J.R. Liu ans L.Z. Fang, *New Astronomy* **13**, 1, (2008)

    We investigate the volume growth of ionized regions around UV photon sources with the WENO algorithm, which is an effective solver of photon kinetics in the phase space described by the radiative transfer equation. We show that the volume growth rate, either of isolated ionized regions or of clustered regions in merging, generally consists of three phases: fast or relativistic growth phase at the early stage, slow growth phase at the later stage, and a transition phase between the fast and slow phases. We also show that the volume growth of ionized regions around clustered sources with intensity $\dot{E}_i$ ($i = 1, 2, ...$) would have the same behavior as a single source with intensity $\dot{E} = \sum_i \dot{E}_i$, if all the distances between nearest neighbor sources i and j are smaller than $c(t_c^i + t_c^j)$, t_c^i being the time scale t_c of source i. Therefore, a tightly clustered UV photon sources would lead to a slow growth of ionized volume. This effect would be important for studying the redshift-dependence of 21cm signals from the reionization epoch.

16. Non-Gaussianity of the Cosmic Baryon Fluid: Log-Poisson Hierarchy Model, J.R. Liu and L. Z. Fang, *Astrophys. J.*, **672**, 11, (2008)

    In the nonlinear regime of cosmic clustering, the mass density field of the cosmic baryon fluid is highly non-Gaussian. It shows different dynamical behavior from collisionless dark matter. Nevertheless, the evolved field of baryon fluid is scale-covariant in the range from the Jeans length to a few ten h^{-1} Mpc, in which the dynamical equations and initial perturbations are scale free. We show that in the scale-free range, the non-Gaussian features of the cosmic baryon fluid, governed by the Navier-Stokes equation in an expanding universe, can be well described by a log-Poisson hierarchical cascade. The log-Poisson scheme is a random multiplicative process (RMP), which causes non-Gaussianity and intermittency even when the original field is Gaussian. The log-Poisson RMP contains two dimensionless parameters: β for the intermittency and γ for the most singular structure. All the predictions given by the log-Poisson RMP model, including the hierarchical relation, the order dependence of the intermittent exponent, the moments, and the scale-scale correlation, are in good agreement with the results given by hydrodynamic simulations of the standard cold dark matter model. The intermittent parameter β decreases slightly at low redshift and indicates that the density field of baryon fluid contains more singular structures at lower redshifts. The applicability of the model is addressed.

17. Lyα Leaks and Reionization,.L. Feng, H.G. Bi, J.R. Liu, and L.Z. Fang, MNRAS, **383**, 1459, (2008)

    Lyα absorption spectra of QSOs at redshifts $z \simeq 6$ show complete Gunn-

Peterson absorption troughs (dark gaps) separated by tiny leaks. The dark gaps are from the intergalactic medium (IGM) where the density of neutral hydrogen are high enough to produce almost saturated absorptions, however, where the transmitted leaks come from is still unclear so far. We demonstrate that leaking can originate from the lowest density voids in the IGM as well as the ionized patches around ionizing sources using semi-analytical simulations. If leaks were produced in lowest density voids, the IGM might already be highly ionized, and the ionizing background should be almost uniform; in contrast, if leaks come from ionized patches, the neutral fraction of IGM would be still high, and the ionizing background is significantly inhomogeneous. Therefore, the origin of leaking is crucial to determining the epoch of inhomogeneous-to-uniform transition of the the ionizing photon background. We show that the origin could be studied with the statistical features of leaks. Actually, Lyα leaks can be well defined and described by the equivalent width W and the full width of half area $W_{\rm H}$, both of which are less contaminated by instrumental resolution and noise. It is found that the distribution of W and $W_{\rm H}$ of Lyα leaks are sensitive to the modeling of the ionizing background. We consider four representative reionization models. It is concluded that the leak statistics provides an effective tool to probe the evolutionary history of reionization at $z \simeq 5 - 6.5$. Similar statistics would also be applicable to the reionization of He II at $z \simeq 3$

18. DWT Analysis of the 2-degree Field Galaxy Redshift Survey, Y.-C. Cai, J. Pan, L.L. Feng and L.Z. Fang, *ChJAA*, **8**, 159, (2008)

    The power spectrum of the two-degree Field Galaxy Redshift Survey (2dFGRS) sample is estimated with the discrete wavelet transform (DWT) method. The DWT power spectra within $0.04 < k < 2.3h\text{Mpc}^{-1}$ are measured for three volume-limited samples defined in connective absolute magnitude bins $-19 \sim -18$, $-20 \sim -19$ and $-21 \sim -20$. We show that the DWT power spectrum can effectively distinguish ΛCDM models of $\sigma_8 = 0.84$ and $\sigma_8 = 0.74$. We adopt maximum likelihood method to perform three-parameter fitting with bias parameter b, pairwise velocity dispersion σ_{pv} and redshift distortion parameter $\beta = \Omega_m^{0.6}/b$ to the measured DWT power spectrum. Fitting results denotes that in a $\sigma_8 = 0.84$ universe the best fitted Ω_m given by the three samples are consistent in the range $0.28 \sim 0.36$, and the best fitted σ_{pv} are 398^{+35}_{-27}, 475^{+37}_{-29} and 550 ± 20km/s for the three samples, respectively. However in the model of $\sigma_8 = 0.74$, our three samples give very different values of Ω_m. We repeat the fitting by using empirical formula of redshift distortion. The result of the model of low σ_8 is still poor, especially, one of the best value σ_{pv} is as large as 10^3km/s. The power spectrum of 2dFGRS seems in disfavor of models with low amplitude of density fluctuations.

19. Scaling relation between Sunyaev-Zel'dovich effect and X-ray luminosity and scale-free evolution of cosmic baryon field, Q. Yan, H.Y. Wan, T.J. Zhang, J.R. Liu, L.L. Feng and L.Z. Fang, *New Astronomy*, **14** 152, (2009)

    It has been revealed recently that, in the scale free range, i.e. from the scale of the onset of nonlinear evolution to the scale of dissipation, the velocity and mass density fields of cosmic baryon fluid are extremely well described by the self-similar log-Poisson hierarchy. As a consequence of this evolution, the relations among various physical quantities of cosmic baryon fluid should be scale invariant, if the physical quantities are measured in cells on scales larger than the dissipation scale, regardless the baryon fluid is in virialized dark halo, or in pre-virialized state. We examine this property with the relation between the Compton parameter of the thermal Sunyaev-Zel'dovich effect, $y(r)$, and X-ray luminosity, $L_{\rm x}(r)$, where r being the scale of regions in which y and $L_{\rm x}$ are measured. According to the self-similar hierarchical scenario of nonlinear evolution, one should expect that 1.) in the $y(r)$-$L_x(r)$ relation, $y(r) = 10^{A(r)}[L_{\rm x}(r)]^{\alpha(r)}$, the coefficients $A(r)$ and $\alpha(r)$ are scale-invariant; 2.) The relation $y(r) = 10^{A(r)}[L_{\rm x}(r)]^{\alpha(r)}$ given by cells containing collapsed objects is also available for cells without collapsed objects, only if r is larger than the dissipation scale. These two predictions are well established with a scale decomposition analysis of observed data, and a comparison of observed $y(r)$-$L_x(r)$ relation with hydrodynamic simulation samples. The implication of this result on the characteristic scales of non-gravitational heating is also addressed.

20. Log-Poisson hierarchical clustering of cosmic neutral hydrogen and Lyα transmitted flux of QSO absorption spectrum, Y. Lu, Y.Q. Chu, and L. Z. Fang, *Astrophys. J.* **691**, 43, (2009)

    We study, in this paper, the non-Gaussian features of the mass density field of neutral hydrogen fluid and the Ly transmitted flux of QSO absorption spectrum from the point-of-view of self-similar log-Poisson hierarchy. It has been shown recently that, in the scale range from the onset of nonlinear evolution to dissipation, the velocity and mass density fields of cosmic baryon fluid are extremely well described by the She-Leveques scaling formula, which is due to the log-Poisson hierarchical cascade. Since the mass density ratio between ionized hydrogen to total hydrogen is not uniform in space, the mass density field of neutral hydrogen component is not given by a similar mapping of total baryon fluid. Nevertheless, we show, with hydrodynamic simulation samples of the concordance ΛCDM universe, that the mass density field of neutral hydrogen, is also well described by the log-Poisson hierarchy. We then investigate the field of Ly transmitted flux of QSO absorption spectrum. Due to redshift distortion, Ly transmitted flux fluctuations are no longer to show all features of the log-Poisson hierarchy. However, some non- Gaussian features predicted by the log-Poisson hierarchy are not affected by the redshift distortion. We test these predictions with the high resolution and high S/N data of quasars Ly absorption spectra. All results given by real data, including -hierarchy, high order moments and scale-scale correlation, are found to be well consistent with the log-Poisson hierarchy. We compare the log-Poisson hierarchy with the popular log-normal model of the Ly transmitted flux. The later is found to yield too strong non-Gaussianity at high orders, while the log- Poisson hierarchy is in agreement with observed data.

21. Time evolution of Wouthuysen-Field coupling, I. Roy, W. Xu, J.M. Qiu, C.W. Shu and L.Z. Fang, *Astrophys. J.* **694** 1121, (2009).

    We study the Wouthuysen-Field coupling at early universe with numerical solutions of the integrodifferential equation describing the kinetics of photons undergoing resonant scattering. The numerical solver is developed based on the weighted essentially non-oscillatory (WENO) scheme for the Boltzmann-like integrodifferential equation. This method has perfectly passed the tests of analytic solution and conservation property of the resonant scattering equation. We focus on the time evolution of the Wouthuysen-Field (W-F) coupling in relation to the 21 cm emission and absorption at the epoch of reionization. We especially pay attention to the formation of the local Boltzmann distribution, $e^{(\nu-\nu_0)/kT}$, of photon frequency spectrum around resonant frequency ν_0. We show that a local Boltzmann distribution will be formed if photons with frequency 0 have undergone a ten thousand or more times of scattering, which corresponds to the order of 10^3 yrs for neutral hydrogen density of the concordance ΛCDM model. The time evolution of the shape and width of the local Boltzmann distribution actually doesn't dependent on the details of atomic recoil, photon sources, or initial conditions very much. However, the intensity of photon flux at the local Boltzmann distribution is substantially time-dependent. The time scale of approaching the saturated intensity can be as long as $10^5 - 10^6$ yrs for typical parameters of the ΛCDM model. The intensity of the local Boltzmann distribution at time less than 105 yrs is significantly lower than that of the saturation state. Therefore, it may not be always reasonable to assume that the deviation of the spin temperature of 21 cm energy states from cosmic background temperature is mainly due to the W-F coupling if first stars or their emission/absorption regions evolved with a time scale equal to or less than Myrs. Subject headings

22. A WENO algorithm for radiative transfer with resonant scattering and the Wouthuysen-Field Coupling, I. Roy, J.M. Qiu, C.W. Shu and L.Z.

Fang, *New Astronomy*, **14**, 513, (2009)

We develop a numerical solver for the integral-differential equations, which describe the radiative transfer of photon distribution in the frequency space with resonant scattering of Lyα photons by hydrogen gas in the early universe. The time-dependent solutions of this equation is crucial to the estimation of the effect of the Wouthuysen-Field (WF) coupling in relation to the 21 cm emission and absorption at the epoch of reionization. However, the time-dependent solutions of this equation have not yet been well performed. The resonant scattering leads to the photon distribution in the frequency space to be piecewise smooth containing sharp changes. The weighted essentially nonoscillatory (WENO) scheme is suitable to handle this problem, as this algorithm has been found to be highly stable and robust for solving Boltzmann equation. We test this numerical solver by 1.) the analytic solutions of the evolution of the photon distribution in rest background; 2.) the analytic solution in expanding background, but without resonant scattering; 3.) the formation of local Boltzmann distribution around the resonant frequency with the temperature to be the same as that of atom for recoil. We find that the evolution of the photon distribution due to resonant scattering with and without recoil generally undergoes three phases. First, the profile of the photon distribution is similar to the initial one. Second, an extremely flat plateau (without recoil) or local Boltzmann distribution (with recoil) form around the resonant frequency, and the width and height of the flat plateau or local Boltzmann distribution increase with time. Finally, the distribution around the resonant frequency is saturated when the photons from the source is balanced by the redshift of the expansion. This result indicates that the onset of the W-F coupling should not be determined by the third phase, but by the time scale of the second phase. We found that the time scale of the W-F coupling is equal to about a few hundreds of the mean free flight time of photons with resonant frequency, and it basically is independent of the Sobolev parameter if this parameter is much less than 1.

23. Wouthuysen-Field coupling in 21 cm region around high redshift sources, I. Roy, W. Xu, J.-M. Qiu, C.-W. Shu, and L. Z. Fang, *Astrophys. J.*, **703**, 1992, (2009)

The 21 cm emission and absorption from gaseous halos around the first generation of stars substantially depend on the Wouthuysen-Field (W-F) coupling, which relates the spin temperature with the kinetic temperature of hydrogen gas via the resonant scattering between Lyα photons and neutral hydrogen. Therefore, the existence of Ly photons in the 21 cm region is essential. Although the center object generally is a strong source of Lyα photons, the transfer of Ly photons in the 21 cm region is very inefficient, as the optical depth of Lyα photons is very large. Consequently, the Ly photons 0 from the source may not be able to transfer to the entire 21 cm region timely to provide the W-F coupling. This problem is especially important considering that the lifetime of first stars generally is short. We investigate this problem with the numerical solution of the integrodifferential equation, which describes the kinetics of Lyα resonant photons in both physical and frequency spaces. We show that the photon transfer process in the physical space is actually coupled to that in the frequency space. First, the diffusion in the frequency space provides a shortcut for the diffusion in the physical space. It makes the mean time for the escape of resonant photon in optical depth τ media roughly proportional to the optical depth τ, not τ^2. Second and more importantly, the resonant scattering is effective in bouncing photons with frequency $\nu \neq \nu_0$ back to ν_0. This process can quickly restore 0 photons and establish the local Boltzmann distribution of the photon spectrum around 0. Therefore, the mechanism of escape via shortcut plus bounce back enables the W-F coupling to be properly realized in the 21 cm region around first stars. This mechanism also works for photons injected into the 21 cm region by redshift.

24. The zeroth Law of thermodynamics of photon-hydrogen system and 21 cm cosmology, L. Z. Fang, *Int. J. of Mod. Phys. D*, **18**, 1943, (2009)

    A basic physical problem of the 21 cm cosmology is the so-called Wouthuysen-Field coupling, which assume that the resonant scattering of Lyα photons with neutral hydrogen atoms will lock the color temperature of the photon spectrum around the Lyα frequency to be equal to the kinetic temperature of hydrogen gas. This assumption actually is the zeroth thermodynamic law on the formation of local statistically thermal equilibrium state of the photon-atom system. However, the time-dependent process of approaching to a local statistically thermal equilibrium with the kinetic temperature has never been studied, as it needs to solve an integral-differential equations - the radiative transfer equation of the resonant scattering. Recently, with a state-of-the-art numerical method, the formation and evolution of the Wouthuysen-Field coupling has been systematically studied. This paper is to review the physical results, including the time scales of the onset of Wouthuysen-Field coupling, the profile of frequency distribution of photons in the state of local thermal equilibrium, the effects of the expansion of the universe, the Wouthuysen-Field coupling in a optical thick halos etc.

25. A wavelet-Galerkin algorithm of the E/B decomposition of CMB polarization maps, L. Cao and L. Z. Fang, *Astrophys. J.*, **706**, 1545, (2009)

    We develop an algorithm of separating the E and B modes of the CMB polarization from the noisy and discretized maps of Stokes parameter Q and U in a finite area. A key step of the algorithm is to take a wavelet-Galerkin discretization of the differential relation between the E, B and Q, U fields. This discretization allows derivative operator to be represented by a matrix, which is exactly diagonal in scale space, and narrowly banded in spatial space. We show that the effect of boundary can be eliminated by dropping a few DWT modes located on or nearby the boundary. This method reveals that the derivative operators will cause large errors in the E and B power spectra on small scales if the Q and U maps contain Gaussian noise. It also reveals that if Q and U maps are random fields, their variances will lead to the cross correlation between the E- and B-modes. Consequently, the B mode will be contaminated if the powers of E modes are much larger than that of B modes. Nevertheless, with reasonable samples, numerical tests show that the power spectra of both E and B on scales that are larger than the finest scale by a factor of 4 and higher can be recovered, even when the power ratio of E- to B-modes is as large as about 10^2, and the signal-to-noise ratio is equal to 10 and higher. This is because 1.) the DWT power spectrum is an effective tool to denoise; the errors rapidly decrease with the increasing of scales; 2.) the Galerkin discretization is free of false correlations, and keeps the contamination under control. As wavelet variables contain information of both spatial and scale spaces, the developed method is also effective to recover the spatial structures of the E and B mode fields.

26. Vorticity of Intergalactic Medium Velocity Field on Large Scales W.S. Zhu, L.L. Feng and L.Z. Fang, *Astrophys. J.*, **712**, 1, (2010)

    We investigate the vorticity of the intergalactic medium (IGM) velocity field on large scales with cosmological hydrodynamic simulation of the concordance model of ΛCDM. We show that the vorticity field is significantly increasing with time as it can effectively be generated by shocks and complex structures in the IGM. Therefore, the vorticity field is an effective tool to reveal the nonlinear behavior of the IGM, especially the formation and evolution of turbulence in the IGM. We find that the vorticity field does not follow the filament and sheet structures of the underlying dark matter density field and shows highly non-Gaussian and intermittent features. The power spectrum of the vorticity field is then used to measure the development of turbulence in Fourier space. We show that the relation between the power spectra of vorticity and velocity fields is perfectly in agreement with the prediction of a fully developed homogeneous and isotropic turbulence in the scale range from 0.2 to about 3 h^{-1} Mpc at $z \sim 0$. This indicates that the cosmic baryonic field is in the state of fully developed turbulence on scales less than about 3 h^{-1} Mpc. The random field of the turbulent fluid yields turbulent pressure to prevent the gravitational collapsing of the IGM. The vorticity and turbulent pressure are strong inside and even outside of high density regions. In IGM regions with 10 times mean overdensity, the turbulent pressure can be on an average equivalent to the thermal pressure of the baryonic gas with a temperature of 1.0×10^5 K. Thus, the fully developed turbulence would prevent the baryons in the IGM from falling into the gravitational well of dark matter halos. Moreover, turbulent pressure essentially is dynamical and non-thermal, which makes it different from a pre-heating mechanism, as it does not affect the thermal state and ionizing process of hydrogen in the IGM.

27. Resonant Scattering and Lyα Radiation Emergent from Neutral Hydrogen Halos, I. Roy, Ishani, C.-W. Shu and L.Z. Fang, *Astrophys. J.*, **716**, 604, (2010)

    With a state-of-the-art numerical method used for solving the integral-differential equation of radiative transfer, we investigate the flux of the Lyα photon ν_0 emergent from an optically thick halo containing a central light source. Our focus is on the time-dependent effects of the resonant scattering. We first show that the frequency distribution of photons in the halo is quickly approaching a locally thermalized state around the resonant frequency, even when the mean intensity of the radiation is highly time dependent. Since initial conditions are forgotten during the thermalization, some features of the flux, such as the two-peak structure of its profile, are actually independent of the intrinsic width and time behavior of the central source, if the emergent photons are mainly from photons in the thermalized state. In this case, the difference $|\nu_\pm - \nu_0|$, where $\nu_\pm$ are the frequencies of the two peaks of the flux, cannot be less than 2 times the Doppler broadening. We then study the radiative transfer in the case where the light emitted from the central source is a flash. We calculate the light curves of the flux from the halo. It shows that the flux is still a flash. The time duration of the flash for the flux, however, is independent of the original time duration of the light source but depends on the optical depth of the halo. Therefore, the spatial transfer of resonant photons is a diffusion process, even though it is not a purely Brownian diffusion. This property enables an optically thick halo to trap and store thermalized photons around ?0 for a long time after the cessation of the central source emission. The photons trapped in the halo can yield delayed emission, of which the profile also shows typical two-peak structure as that from locally thermalized photons. Possible applications of these results are addressed.

28. Log-Poisson non-Gaussianity of Lyα transmitted flux fluctuations at high redshift, Y. Lu, W.S. Zhu, Y.Q. Chu, L.L. Feng and Fang, Li-Zhi, MNRAS, **408**, 452, (2010)

    We investigate the non-Gaussian features of the intergalactic medium

(IGM) at redshift z 5-6 using the Lyα transmitted flux of quasar absorption spectra and a cosmological hydrodynamic simulation of the concordance ΛCDM universe. We show that the neutral hydrogen mass density field and Lyα transmitted flux fluctuations possess all the non-Gaussian features predicted by the log-Poisson hierarchy. This depends only on two dimensionless parameters α and β, describing, respectively, the intermittence and singularity of the random fields. We find that the non-Gaussianity of the Lyα transmitted flux of quasars from z = 4.9 to z = 6.3 can be well reconstructed by the hydrodynamical simulation samples. Although the Gunn-Peterson optical depth and its variance undergoes a significant evolution in the redshift range of 5-6, the intermittency measured by β is almost redshift-independent in this range. More interestingly, the intermittency of the quasar's absorption spectra on physical scales 0.1-1 h^{-1} Mpc in redshift 5-6 is found to be about the same as that on physical scales 1-10 h^{-1} Mpc at redshifts 2-4. Considering the Jeans length is less than 0.1 h^{-1} Mpc at z 5, and 1 h^{-1} Mpc at z 2, these results imply that the non-linear evolution in high and low redshifts will lead the cosmic baryon fluid to a state similar to fully developed turbulence. The log-Poisson high-order behaviour of the current high-redshift data of a quasar's spectrum can be explained by the uniform ultraviolet background in the redshift range considered. We have also studied the log-Poisson non-Gaussianity by considering an inhomogeneous background. With several simplified models of the inhomogeneous background, we have found that the effect of the inhomogeneous background on the log-Poisson non-Gaussianity is no larger than 1σ.

29. Statistical and dynamical decoupling of the IGM from Dark Matter L.Z. Fang, W. Z. Zhu, Advances in Astronomy, **2011**, Article ID 492980, 9 pages, (2011)

    Although the gravitational field in the universe is dominated by dark matter, cosmological observations show that the statistical properties of cosmic baryonic matter are significantly and systematically decoupled from that of the underlying dark matter. The dynamical reason of the decoupling is the difference of the nonlinear evolution of baryon fluid from that of collisionless dark matter. In highly nonlinear regime, the cosmic baryon fluid on scale free range evolves into the state of fully developed turbulence, of which the velocity field consists of shocks, vortices and complex structures. This scenario provides a coherent explanation of various phenomena referring to the statistical and dynamical decoupling of the IGM from dark matter.

Book Chapters and Proceedings

1. Intergalactic medium in the LCDM universe from cosmological simulations, L.L. Feng, P. He, L.Z. Fang, C.W. Shu and M.P. Zhang, *J. of Korean Astr. Soc.*, **38**, 129, (2005)

2. Long-tailed time-dependent correlation of primordial cosmic perturbations in the inflationary cosmology and its observable effects, L.Z. Fang, in *Inquiring the Universe: Essays to celebrate Professor Mario Novellon jubilee*, Frontier Groups, (ISBN 2914601085) (2005)

3. The history of reionization, L. Z. Fang, *J. of Korean Phys. Soc.*, **49**, 697, (2006)

4. The DWT power spectrum of the two-degree field galaxy redshift survey, Y.C. Cai, J. Pan, Y.H. Zhao, L.L. Feng and L.Z. Fang, *Relativistic Astrophysics* eds. C.L. Bianco and S.-S. Xue, the AIP Conference Proceedings, **966**, 87, (2007)

5. Twenty one cm signals from ionized and heated regions around first stars, L.Z. Fang, *Relativistic Astrophysics* eds. C.L. Bianco and S.-S. Xue, the AIP Conference Proceedings, **966**, 95, (2007)

6. Intermittency of cosmic baryon fluid, L.Z. Fang, *Relativistic Astrophysics* eds. D.S. Lee, W.L. Lee and S.-S. Xue, the AIP Conference Proceedings, in press (2008)

Invited talks at international conferences

1. Colloquium: Turbulence-like behavior of cosmic baryon fluid and cosmological hydrodynamical simulation, Brown University, October 28, 2005

2. Public talk: When would the sky fall ? July 20, 2005 Seoul C Mt. Kumgang, Korea

3. Invited talk: Abnormal features of cosmic temperature fluctuations. 14 June 2005, Pescara, Italy

4. Invited talk: Einstein, Social responsibility of physicists and human rights in China, APS March meeting, March 22, 2005, Los Angeles.

5. Invited talk: chronology of the dark ages of the universe, June 20, 2005, Seoul-Mt. Kumgang, Korea

6. Invited talk: Stories of SN 1006 in ancient Chinese literatures. 11 July 2006, meeting on "Supernova, GRB and cosmology" Pescara, Italy

7. Invited talk: Non-linear evolution of cosmic baryon fluid 13 July 2006, on "Supernova, GRB and cosmology" Pescara, Italy

8. Invited talk: Scaling in Cosmology, Institute of Physics, Academia Sinica, Taipei, May 30, 2007
9. Invited talk: 21 cm signals from ionized and heated regions around first stars, 4th Italian-Sino workshop on relativistic astrophysics, 23 July 2007
10. Invited talk: The standard cosmological model, Taipei School/Workshop on Large Scale Structures of the Universe National Center for Theoretical Sciences, May 28 C June 2, 2007
11. Invited talk: primordial perturbations, Taipei School/Workshop on Large Scale Structures of the Universe National Center for Theoretical Sciences, May 28 C June 2, 2007
12. Invited talk: nonlinear evolution of intergalactic medium (IGM), Taipei School/Workshop on Large Scale Structures of the Universe National Center for Theoretical Sciences, May 28 C June 2, 2007
13. Invited talk: probe of dark energy with large scale structures, Taipei School/Workshop on Large Scale Structures of the Universe National Center for Theoretical Sciences, May 28 C June 2, 2007
14. Colloquium: Studying cosmic baryon fluid with cosmological hydrodynamic simulation, NY university at Stony Brook, Feb 20, 2008
15. Colloquium: Studying cosmic baryon fluid with hydrodynamic simulation, Institute of Physics, Academia Sinica, Taipei, May 27, 2008
16. Invited talk: Intermittency of cosmic baryon fluid, 5th meeting on relativistic astrophysics, Taipei, May 28, 2008
17. Invited talk: Intermittency of cosmic baryon fluid, 5th Italian-Sino workshop on relativistic astrophysics, May 29, 2008
18. Public talk: 21 cm signals from early universe, DongHwa University, HuaLien, June 2, 2008
19. Colloquium: A fundamental physical problem in 21 cm cosmology, University of Texas at Arlington, April 1, 2009
20. Colloquium: The zeroth law of thermodynamics of photon-atom system and 21 cm cosmology, University of Taiwan, June 16, 2009
21. Invited talk: Wouthuysen-Field coupling, 6th Italian-Sino workshop on relativistic astrophysics, Pescara, June 29, 2009
22. Nonlinear evolution of cosmic baryon matter, 2nd Galileo-XuGuangqi Meeting, Ventimiglia, July 12, 2010